A Method of Treating the Nongray Error in Total Emittance Measurements

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Introduction

IN techniques presently available for the rapid determination of total emittance, the sample is generally exposed to surroundings that are at a different temperature than the sample's surface. When the infrared spectral reflectance of the surface is spectrally selective, an error is introduced into the total emittance values so determined. It has been shown that highly reflecting aluminum surfaces overcoated with dielectric films of varying thickness can produce coatings that have a wide range of emittance (ϵ) values.[1,2] These dielectric films are relatively nonabsorbing in the solar region but rely on a combination of interference and absorption effects to give high infrared emittances. The solar absorptance is essentially independent of thickness, whereas ϵ increases steadily with increasing thickness of the dielectric film. For a certain range of thickness, ϵ is strongly thickness-dependent so that a slight change in thickness can cause a rather large change in emittance. This means that the thickness must be carefully controlled and monitored. Surfaces of this type exhibit a spectrally varying infrared reflectance as a result of the combination of absorption and interference effects. This spectral selectivity of the infrared reflectance gives rise to the nongray error that exists in devices commonly used for the rapid determination of total emittance, and it has been shown that this error can be quite large.[3] Although it is possible in many cases to eliminate this error through proper calibration with samples of known emittance, when the emittance is thickness-dependent, calibration is either impossible or impractical. Therefore, the magnitude of the error must be determined for the type of device used.

Experimental Techniques

Total emittance measurements were performed in this investigation with a Gier-Dunkle Model DB-100 portable infrared reflectometer. This device produces a weighted value of the reflectance of an opaque sample from which the emittance is determined. The measurement is performed relative to high- and low-reflectance standards. It is instantaneous and independent of surface temperature over the range 20–60°C. The operating principles and design details of a prototype of this device have been presented in the literature.[4] The values of relative weighted reflectance are obtained from hohlraum measurements. Any surface covering the sample port is alternately exposed to omnidirectional radiation emitted by hot and cold semicylinders acting as blackbody cavities. Since the illuminated area is viewed by the detector in a near-normal direction, all parameters dealt within the following analysis are considered to be averaged over the solid angle subtended by this area. For the infrared region considered here and for the narrow viewing angle used in this device, it is safe to say that

$$\rho = \rho(\theta) \text{ and } \epsilon = \epsilon(\theta)$$

When a test surface covers the viewing port of the instrument's sensing head and is irradiated by the rotating semicylindrical cavities, an alternating signal is produced in the detector due to the fact that the cavities are at different temperatures. The intensity of radiant flux coming from the direction of the sample when irradiated by the hotter cavity is

$$I_{SH} = \epsilon_S(T_S)\tau(T_S)\sigma T_S^4 + \rho_S(T_S,T_H)\tau(T_S,T_H)\sigma T_H^4 + C \quad (1)$$

where $\epsilon_S(T_S)$ = near-normal emittance for a sample at temperature T_S; $\rho_S(T_S,T_H)$ = near-normal reflectance for a sample at temperature T_S when irradiated by the surrounding cavity walls at T_H; $\tau(T_S,T_H)$ = transmittance of a polyethelene compensating filter used to modify the spectral distribution of radiant energy reaching the detector; σ = Stefan-Boltzmann constant; and C = a term used to include all other sources of emitted or reflected energy reaching the detector.

Similarly, when the surface is exposed to the colder cavity,

$$I_{SC} = \epsilon_S(T_S)\tau(T_S)\sigma T_S^4 + \rho_S(T_S,T_C)\tau(T_S,T_C)\sigma T_C^4 + C \quad (2)$$

Equations (1) and (2) show that only the reflected energy varies with this alternate irradiation by the two semicylinders. The detector amplifying system is made to respond only to the alternating signal arising in the reflectance terms[4] so that the fluctuating portion of the signal is the difference between Eqs. (1) and (2). That is,

$$KV_S = I_{SH} - I_{SC} = \rho_S(T_S,T_H)\tau(T_S,T_H)\sigma T_H^4 - \rho_S(T_S,T_C)\tau(T_S,T_C)\sigma T_C^4 \quad (3)$$

where the voltage V_S of the output signal is proportional (K) to the energy difference. Equation (3) represents the case for an unknown sample covering the opening of the sensing head. It is necessary to calibrate the instrument by establishing known voltage levels to define the range of the output signal. This is done using high- and low-reflectance standards resulting in the following intensity equations for each case:

$$KV_{100} = \rho_{100}(T_S,T_H)\tau(T_S,T_H)\sigma T_H^4 - \rho_{100}(T_S,T_C)\tau(T_S,T_C)\sigma T_C^4 \quad (4)$$

$$KV_0 = \sigma_0(T_S,T_H)\tau(T_S,T_H)\sigma T_H^4 - \rho_0(T_S,T_C)\tau(T_S,T_C)\sigma T_C^4 \quad (5)$$

Here $\rho_{100}(T_S,T_H)$ and $\rho_0(T_S,T_H)$ are the reflectances of the high and low standards, respectively, and V_{100} and V_0 are the corresponding voltages of the output signal.

If the high- and low-reflectance standards have no spectral variation in reflectance over the wavelength range specified by the planckian distributions for σT_H^4, σT_C^4, and σT_S^4, and the surface properties of the materials are invariant over the temperature range of T_H, T_C, and T_S, then the references are gray reflectors.[5]

When $\rho_{100}(T_S)$ and $\rho_0(T_S)$ are accurately known from independent measurements, the voltages V_{100} and V_0 can be proportionately scaled so that $\rho_{100}(T_S) \simeq 100\%$ and $\rho_0(T_S) \simeq 0\%$. The measured reflectance is then given by:

$$\rho_S(T_S,T_H,T_C) = \frac{[\rho_S(T_S,T_H)\tau(T_S,T_H)\sigma T_H^4 - \rho_S(T_S,T_C)\tau(T_S,T_C)\sigma T_C^4]}{[\tau(T_S,T_H)\sigma T_H^4 - \tau(T_S,T_C)\sigma T_C^4]} \quad (6)$$

To include the more general nongray case, the parameters of Eq. (6) are replaced by their wavelength dependent equivalents. Equation (6) then becomes

$$\rho_S(T_S,T_H,T_C) = \left[\int_0^\infty \rho_\lambda(T_S)\tau_\lambda(T_S)E_\lambda(T_H)d\lambda - \int_0^\infty \rho_\lambda(T_S)\tau_\lambda(T_S)E_\lambda(T_C)d\lambda\right] \Big/ \left[\int_0^\infty \tau_\lambda(T_S)E_\lambda(T_H)d\lambda - \int_0^\infty \tau_\lambda(T_S)E_\lambda(T_C)d\lambda\right] \quad (7)$$

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where

$$\sigma T^4 = \int_0^\infty E_\lambda(T)d\lambda$$

and $E_\lambda(T)$ is the planckian spectral irradiance function for a blackbody at temperature T.

Equation (7) can be written

$$\rho_S(T_S,T_H,T_C) = \left\{\int_0^\infty \rho_\lambda(T_S)\tau_\lambda(T_S)[E_\lambda(T_H) - E_\lambda(T_C)]d\lambda\right\} \Big/ \left\{\int_0^\infty \tau_\lambda(T_S)[E_\lambda(T_H) - E_\lambda(T_C)]d\lambda\right\} \quad (8)$$

This shows that the measured reflectance is equal to the integrated true spectral reflectance modified by the indicated weighting function. By definition, the true weighted reflectance of a surface at temperature T_S is given by

$$\rho_S(T_S) = \int_0^\infty \rho_\lambda(T_S)E_\lambda(T_S)d\lambda \Big/ \int_0^\infty E_\lambda(T_S)d\lambda \quad (9)$$

The difference between the true reflectance and the measured reflectance is the measurement error, which can be defined as

$$\delta\rho_S = \rho_S(T_S) - \rho_S(T_S,T_H,T_C) \quad (10)$$

A summation technique is used to evaluate the above integrals. Equation (8) may be rewritten to give

$$\rho_S(T_S,T_H,T_C) = \sum_{n=1}^{m} \rho_{\Delta\lambda n}(T_S)q_n$$

where

$$q_n = \{\tau_{\Delta\lambda n}(T_S)[E_{\Delta\lambda n}(T_H) - E_{\Delta\lambda n}(T_C)]\} \Big/ \left\{\sum_{n=1}^{m} \tau_{\Delta\lambda n}(T_S)[E_{\Delta\lambda n}(T_H) - E_{\Delta\lambda n}(T_C)]\right\}$$

Here $\rho_{\Delta\lambda n}$, $\tau_{\Delta\lambda}$, and $E_{\Delta\lambda}$ are defined over a wavelength band $\Delta\lambda$ rather than at a discreet wavelength λ. The summation is over the range $0 \leq \lambda \leq \infty$ and consequently, if m is large, each $\Delta\lambda n$ will be small. Accuracy dictates the choice of $\Delta\lambda$, since the summation must follow the wavelength variation of each parameter in the integral. In a similar manner, Eq. (9) may be written

$$\rho_S(T_S) = \sum_{n=1}^{m} \rho_{\Delta\lambda n}(T_S)p_n$$

where $p_n = E_{\Delta\lambda n}/\sigma T_S^4$.

It is now possible to express the measurement error defined

Table 1 Near-normal total emittance[a] at 300°K

Sample	Oxide thickness (μm)	Determined directly: Uncorrected	Determined directly: Corrected	Deduced from spectral measurements
Evaporated Al coated with reactively deposited silicon oxide	0.17	0.013	0.011	0.013
	0.36	0.030	0.028	0.025
	1.11	0.245	0.221	0.222
	1.48	0.385	0.360	0.359
	1.94	0.455	0.443	0.445
	2.32	0.510	0.505	0.525
	3.06	0.555	0.558	0.583
ALZAK	2.5	0.680	0.682	0.69
	5.0	0.73	0.75	0.78

[a] The third decimal place in the total emittance data is given only to il- the trend of the correction and is indicative of the reproducibility the accuracy of the measurements.

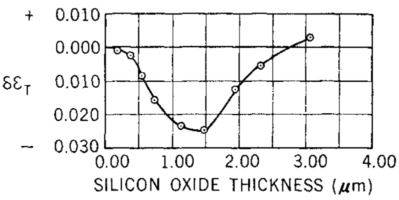

Fig. 1 Calculated error in total emittance as a function of silicon oxide thickness.

by Eq. (10) in terms of the summations presented above;

$$\delta\rho_S = \sum_{n=1}^{m} \rho_{\Delta\lambda n}(T_S)(p_n - q_n) \quad (11)$$

From Kirchhoff's law it follows that

$$\delta\epsilon_S(T_S) = -\delta\rho_S(T_S) \quad (12)$$

for an opaque surface.

p_n and q_n were each calculated. Equations (8) and (11) show that the measurement error is contained within the difference between the two weighting functions, and if they were equal there would be no error. The presence of the term $\tau_\lambda(T_S)$ is justified by the fact that it reduces the difference between the two weighting functions. In most cases the surface whose emittance is to be determined is at room temperature, so that $T_S \simeq 300°K$. The cavity temperatures are controlled at $T_H \simeq 315°K$ and $T_C \simeq 305°K$. The difference between $E_\lambda(300°K)$ and $[E_\lambda(315°K) - E_\lambda(305°K)]$ is largest at the shorter wavelengths, so if a selective filter is used this difference can be minimized. From Eq. (11) it is evident that $\delta\rho_S$ is large for samples having high reflectances at those wavelengths where $(p_n - q_n)$ is large. If the spectral reflectance is unknown beforehand, as is generally the case, a significant error can be recorded.

Since p_n and q_n are normalized functions whose integrals are unity, the sum of their differences is zero. That is,

$$\sum_{n=1}^{m} (p_n - q_n) = 0$$

Therefore, when a surface has a reflectance that is invariant with wavelength and $\rho_{\Delta\lambda n}(T_S)$ is a constant in Eq. (17), then $\delta\rho_S = 0$.

Results

Practical examples of how the nongray error influences total emittance measurements are given in Table 1. Infrared spectral reflectance data were obtained for evaporated aluminum samples coated with various thicknesses of reactively deposited silicon oxide and also for Alzak, a coating produced by an anodic deposition of aluminum oxide onto an aluminum surface.[6] Both materials exhibit thickness-dependent infrared spectral selectivity and have been used as satellite temperature control surfaces. The magnitude of the nongray error was calculated from Eqs. (9, 11, and 12) and is shown in Fig. 1 for the silicon oxide coated aluminum samples. Because the magnitude of the error is thickness-dependent, it is not practical to estimate it by calibrating the measuring device with another surface of known thickness and emittance.

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Observations on the Thermally Induced Twist of Thin-Walled Open-Section Booms

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Nomenclature

$BM_x(z)$ = $e_c Eh \int_0^p \tilde{T}(s,z)\cdot Y_c(s)\cdot ds$, the thermal bending moment about X axis
$BM_y(z)$ = $e_c Eh \int_0^p \tilde{T}(s,z)\cdot X_c(s)\cdot ds$ the thermal bending moment about Y axis
C = torsional rigidity of boom
C_1 = warping rigidity of boom
E = Young's modulus
e_c = coefficient of thermal expansion
$\mathbf{e}_{sc}$ = vector from the shear center to the surface element ds
$f_0(z),f_1(z),f_2(z)$ = functions of z
I_x = $\int_0^p hY_c^2(s)\cdot ds$
I_y = $\int_0^p hX_c^2(s)\cdot ds$
$\tilde{T}(s,z)$ = local temperature
$\tilde{T}_A$ = ambient temperature
$\tilde{T}_m(z)$ = $\frac{1}{p}\int_0^p \tilde{T}(s,z)\cdot ds$, the section averaged temperature
$T_{sc}(z)$ = thermally induced torque
$\epsilon_z(s,z)$ = thermal strain in z direction
$\phi(L)$ = angular response at tip
$\sigma_z(s,z)$ = normal stress in z direction
$\tau(s,z)$ = shear stress in sz plane

Introduction

STRUCTURAL considerations during launch and attitude maneuvers of spacecraft necessitate the use of mechanical subsystems that can be deployed and retracted in orbit. Open sections booms (STEMs)[1] are utilized as communication antennas, gravity gradient stabilizing booms, or as structural components in package transfer devices, solar panel actuators, and instrument carriers. In some applications, it is necessary that the open section boom have minimum twisting, and the various sources including solar heating, that induce twisting have to be identified and evaluated. A particular requirement of this knowledge is for the mass spectrometer experiments intended on Apollo 15 and 16 lunar missions. The mass spectrometer is mounted at the tip of a 25-ft-long BI-STEM† boom and requires a nominal pointing accuracy of $\pm 15°$.

Thermal distortion (bending and twisting) occurs in booms due to the interactions of both temperature distributions and mechanical end conditions. Thermal effects cause the boom to distort to its minimum strain energy position. Extensive work has been carried out by many investigators in predicting both the static and dynamic behavior of thin-walled cylinders of open sections when exposed to solar radiation.

The prerequisite to predicting the steady-state behavior of thin-walled open sections is to model the complex heat-transfer characteristics at the interfaces. This hypothetically, enables the thermoelastic equations to be solved simultaneously with the thermal equilibrium equation. However, the latter equation is nonlinear and further simplifying assumptions have to be made. The work of Florio and Hobbs[2] in solving the thermal equilibrium equation is significant. Subsequently, the work of Graham[3] has been used in solving the more general case which includes heat transfer by internal radiation and interface conduction in overlapped open section booms. The numerical solution of this equation will yield results that are correct to the extent that the assumed modelling is justified. This ambiguity in the results may necessitate experimental investigation to be carried out on the boom. The variations in the lengths of flight booms range from a few meters to 200. In addition, the gravitational problems in simulating the environment of outer space among a host of others restrict the size of the test apparatus.

Experimental investigations thus pose the problem of scaling the results to the desired length of the boom. The objective of this Note is to re-examine the problem of thermally induced twist of open section booms with zero isothermal twist, and present rapid methods of predicting their steady state behavior. Analytical, physical and experimental considerations are utilized for arriving at the values of thermal twist.

Thermally Induced Torque

Restricting the discussion to a circular thin-walled open section boom (Fig. 1), the thermally induced torque $T_{sc}(z)$ may be expressed as

$$\mathbf{T}_{sc}(z) = -\int_0^p \mathbf{e}_{sc} \times h\boldsymbol{\tau}(s,z)\cdot ds \qquad (1)$$

For equilibrium in the thin-walled member the shear stress $\tau(s,z)$ is related to the longitudinal stress $\sigma_z(s,z)$ by the relationship

$$\partial h\cdot\tau(s,z)/\partial s = -h\partial\sigma_z(s,z)/\partial z \qquad (2)$$

The thermo-elastic equation[4] for the stress $\sigma_z(s,z)$ is

$$\sigma_z(s,z) = E\{\epsilon_z(s,z) - e_c[\tilde{T}(s,z) - \tilde{T}_A]\}$$

With the implicit assumption that plane sections remain plane by the expression for $\epsilon_z(s,z)$ that

$$\epsilon_z(s,z) = f_0(z) + f_1(z)X_c(s) + f_2(z)Y_c(s)$$

the relationship for $\sigma_z(s,z)$ may be obtained[5] to satisfy the equilibrium conditions about the principal axes X and Y as

$$\sigma_z(s,z) = E\cdot e_c[\tilde{T}_m(z) - \tilde{T}(s,z)] - [BM_y(z)/I_y]\cdot X_c(s) + [BM_x(z)/I_x]\cdot Y_c(s) \qquad (3)$$

Partially differentiating equation, Eq. (3), with respect to z, we see that for a particular s, the longitudinal distribution of the temperature $\tilde{T}(s,z)$ controls the value of $\partial\sigma_z(s,z)/\partial z$ as $BM_x(z)$ and $BM_y(z)$ are also functions of $\tilde{T}(s,z)$. For a uniform open section boom, whose orientation is constant along

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† BI-STEM: Proprietary name for two slit thin walled cylinders nested with seams in opposition.